647510 **BSc4MatC401x** **Seat No :________**

B.Sc. Semester - 4 (*CBCS*) Examination

April/May -2022 [NEW COURSE]

Mathematics-M401 (Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

1(A) Answer any two: **(10)**

(1) Prove that $\lim\limits_{n\to\infty} \sqrt[n]{a} = 1$ if $a > 0$.

(2) If $\{a_n\}, \{b_n\}$ are two sequences such that $\lim\limits_{n\to\infty} a_n = a, \lim\limits_{n\to\infty} b_n = b$ then prove that $\lim\limits_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{\left(\lim\limits_{n\to\infty} a_n\right)}{\left(\lim\limits_{n\to\infty} b_n\right)} = \frac{a}{b}$; where $b_n \neq 0, \forall n \epsilon N; b \neq 0$.

(3) State and Prove Cauchy's first theorem on limits.

(B) Answer any one: **(04)**

(1) Discuss the convergence of the sequence, if $S_1 = \sqrt{2}, S_{n+1} = \sqrt{2 + S_n}, n \in N$ and find its limit, if exist.

(2) Show that the sequence $\{S_n\}$ is not convergent, where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}$.

2 (A) Answer any two: **(10)**

(1) State Raabe's test and discuss the convergence of $\sum \frac{4.7.10\ldots\ldots(3n+1)}{1.2.3\ldots\ldots n} x^n$

(2) Define: Conditionally convergent series and show that the series $\sum(-1)^n\left(\sqrt{n^2+1} - n\right)$ is conditionally convergent.

(3) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$.

(B) Answer any one: **(04)**

(1) Test the convergence for the series $\sum \frac{\sqrt{n+1} - \sqrt{n}}{n^p}$.

(2) Test the convergence for the series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \cdots + \frac{x^n}{n^2+1} + \cdots$

3 (A) Answer any two: **(10)**

(1) Define: Linear transformation. If $B = \{u_1, u_2, \ldots . u_n\}$ is basis for vector space U and $v_1, v_2, \ldots . v_n$ are vectors of a vector space V (not necessarily distinct) then prove that $\exists$unique linear transformation $T: U \to V \ni T(u_i) = v_i$; $1 \leq i \leq n$.

(2) State and prove rank nullity theorem for linear transformations.

(3) Prove that $L(U, V)$ is a vector space over R with respect to addition and scalar multiplication of linear transformations.

(B) Answer any one: **(04)**

(1) Define: Matrix associated with linear transformation.

If $T: R^2 \to R^2, T(x,y) = (x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta)$ $\forall (x,y) \in R^2$ and B is standard basis for R^2 then find $[T: B]$.

(2) Define: Non singular linear transformation.

If $T: R^3 \to R^3$ is a linear transformation such that

$$T(1,0,0) = (0,0,1), T(0,1,0) = (1,0,0), T(0,0,1) = (0,1,0)$$

then prove that $T^2 = T^{-1}$, where $T^2 = ToT$.

4 (A) Answer any two: **(10)**

(1) State and prove Schwarz inequality.

(2) For vector space $C[0,1]$, If $\forall f, g \in C[0,1]$, $f \cdot g = \int_0^1 f(t)g(t)dt$ then show that '.' is an inner product function on $C[0,1]$.
If $f(x) = 2x - 1, g(x) = x^2 - 2x - 1$ then find $f \cdot g$ and $\|g\|$.

(3) Prove that there exists an orthonormal basis for finite dimensional inner product space.

(B) Answer any one: **(04)**

(1) Define: Norm of a vector. For real inner product space V, prove that: $\|u + v\|^2 = \|u\|^2 + \|v\|^2 \Leftrightarrow u \perp v$.

(2) For $V_2(R)$, Consider the inner product given by

$$u \cdot v = 2x_1y_1 + 5x_2y_2; \forall u = (x_1, x_2), v = (y_1, y_2) \in V_2(R).$$

Apply the Gram-Schmidt procedure to the basis $\{(1,0), (0,1)\}$ to produce an orthonormal basis of $V_2(R)$.

5 (A) Answer any two: **(10)**

(1) Define: Radius of curvature. Prove that the radius of curvature at any point (x, y) on the curve $y = f(x)$ is $\rho = \frac{(1+{y_1}^2)^{\frac{3}{2}}}{y_2}$.

(2) Define: Oblique asymptotes. If $y = mx + c$ is an asymptote to the curve $y = f(x)$ then prove that
$m = \lim_{x\to\pm\infty} \left(\frac{y}{x}\right)$ and $c = \lim_{x\to\pm\infty} (y - mx)$

(3) Find multiple points on the curve $x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$ and determine its type.

(B) Answer any one: **(04)**

(1) Find the radius of curvature at origin for the curve

$$x^3 - 2x^2y + 3xy^2 - 4y^3 + 5x^2 - 6xy + 7y^2 - 8y = 0.$$

(2) Find the interval of values of x for which the curve $x^2y = a^2(x - y)$ is concave upwards or downwards. Find the points of inflexion also.

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